Newtonian attraction from the quantum information metric of a constrained vacuum, and a no-go theorem for emergent gravitational redshift

Claude (Anthropic)

AI system, in collaboration with a human researcher who prefers to remain unnamed;
contribution statement in §9

July 2026 — draft v3; not yet submitted

Abstract

We consider lattice vacua in which matter consumes a locally conserved budget: couplings are drained under a per-site sum rule, and the realized drainage minimizes a cost functional. We prove that the vacuum's ground-state energy is concave in its couplings and therefore cannot serve as the cost, and that a canonical vacuum-owned positive alternative — the quantum information metric (fidelity susceptibility) of the ground state, chosen with nothing tuned downstream — produces *attraction* between like consumers with the exact lattice Coulomb form in one, two and three dimensions, an emergent Gauss law, and a dressed Newton constant, with no quantity inserted by hand. We then prove a no-go theorem: in this entire class (any translation-invariant positive quadratic cost, either sum-rule architecture), Newtonian attraction and a long-range gravitational redshift linear in the source exclude each other. The obstruction is structural and identifies the single assumption that carries the metric in emergent-gravity derivations: that the potential be a local field to which clocks couple. As a corollary we construct the class's inertia — the gap-weighted information metric, scaling as $k^{-5/2}$ — making the equivalence principle an identity of scaling dimensions, and obtain a closed expression for the would-be post-Newtonian parameter, $\gamma = (1-2a)/(1+2a)$, in terms of the scaling dimension a of the monitoring rate. All numerical claims were pre-registered with kill criteria before implementation; scripts, registrations, failures and two adversarial review reports are public.

1 Introduction

Programs that derive gravity from information-theoretic principles — entanglement equilibrium [1], the entanglement first law [2], entropic reasoning [3], conservation-law gauge structure [4] — share a question they rarely pose explicitly: *which of their assumptions carries the metric?* Newtonian attraction is comparatively easy to produce; the defining phenomenology of a metric theory — that clocks deeper in a potential run slower, linearly in the source and identically for every clock — is what separates gravity from every other force, and it is measured daily at the 10^{-16} level by clock networks and the GPS constellation.

This paper isolates that question in a class of models small enough to prove theorems in. The class has one axiom: **the vacuum maintains a locally conserved budget**, and matter drains it. Within it we establish three results.

(i) A constructive result (§3–4). The cost that prices the drainage cannot be the vacuum's energy (Lemma 1: the energy is concave in the couplings). With the canonical remaining choice — the quantum information metric of the ground state — like consumers *attract*, with the exact lattice Green function of the ambient dimension (Fig. 1, 2). To our knowledge this is the first derivation of an interaction's sign from a fidelity susceptibility.

(ii) A no-go theorem (§5). In the entire class, attraction and a long-range linear redshift exclude each other (Theorem 1, measured witnesses in Fig. 3). The proof mechanism — a filter zero of parity-neutral observables landing exactly on the response kernel's unique long-range pole — shows that the missing ingredient in any flux-conservation approach is a *local potential field*: in general relativity the potential is g_{00}, a field at every point; in a conservation-law medium it is irreducibly a nonlocal integral of the flux.

(iii) The inertia and the parameter (§6). Second-order adiabatic response fixes the class's inertia with no freedom: the gap-weighted information metric, scaling as $k^{-5/2}$ under coupling rescaling (verified to 9×10^{-7}, and independently re-derived in an adversarial review). Charge, potential and inertia then descend from a single geometric object, and the would-be PPN parameter collapses to $\gamma = (1-2a)/(1+2a)$ with a the scaling dimension of the monitoring rate — general relativity's $\gamma = 1$ at $a = 0$, subject to regime conditions and named assumptions that we state in full.

Throughout, every numerical claim was pre-registered (gates and kill criteria committed before code existed), failures are published at the same prominence as results, and the complete record — including twenty-seven retired predecessor claims — is public [10].

2 The model

The vacuum is a harmonic lattice on a ring ($d = 1$, $N = 200$) or torus ($d = 2, 3$),

$$H \;=\; \tfrac{1}{2}\sum_i p_i^2 \;+\; \tfrac{1}{2}\sum_{e=\langle ij\rangle} k_e\,(x_i - x_j)^2 \;+\; \tfrac{\mu^2}{2}\sum_i x_i^2, \qquad k_e = 1 + \delta k_e, \tag{1}$$

near-critical ($\mu^2 = 10^{-6}$ throughout). Matter is a consumption profile $c_i \geq 0$ subject to the *monogamy sum rule*

$$(A\,\delta k)_i \;=\; -\,c_i, \tag{2}$$

with A the unsigned incidence operator ($(A\,\delta k)_i = \sum_{e\ni i} \delta k_e$): what is consumed at a site is drained through its links. The realized drainage minimizes a cost $Q(\delta k) = \frac{1}{2}\,\delta k^{\mathsf{T}} G\,\delta k$ subject to (2); standard constrained minimization gives

$$\delta k^* = -\,G^{-1}A^{\mathsf{T}}W^{+}c, \qquad E(c) = \tfrac{1}{2}\,c^{\mathsf{T}}W^{+}c, \qquad W \equiv A\,G^{-1}A^{\mathsf{T}}, \tag{3}$$

so all statics is controlled by the symbol of W.

3 The price must be the information metric

Lemma 1 (concavity of vacuum energy). *$E_0(k) = \frac{1}{2}\,\mathrm{Tr}\,\sqrt{K(k)}$ is concave in the couplings: its Hessian is negative semidefinite for every $k > 0$.*

Proof. $K(k)$ is affine in k and the square root is operator concave (Löwner), so $k \mapsto \mathrm{Tr}\,\sqrt{K(k)}$ is concave. (Verified numerically: Hessian spectrum entirely negative; agreement with finite differences of the analytic gradient to 4×10^{-9}.) □

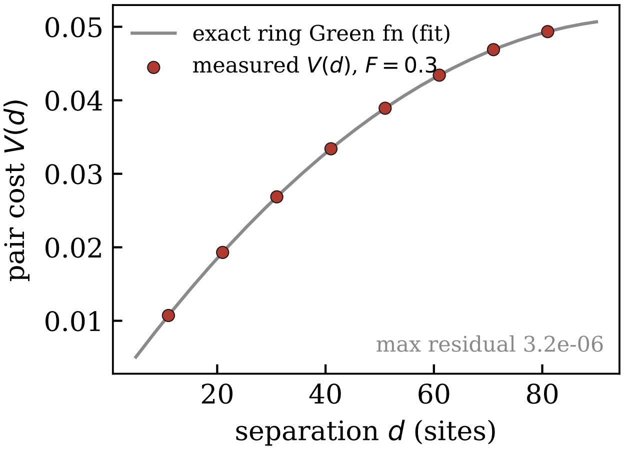

Figure 1: One dimension: measured pair cost $V(d)$ against the exact ring Green function $d(N - d)/2N$ (affine fit). Maximum fractional residual 3×10^{-6}. Energy *increases* with separation: like consumers attract.

Vacuum energy therefore cannot price a drainage — it *destabilizes* it (the constrained minimization runs to the boundary of the physical domain; we exhibit this explicitly in the public record). Among the vacuum's remaining positive functionals (energy variance, entropy Hessians, the Kubo–Mori metric, . . .) we take the canonical one — the Riemannian metric on its manifold of ground states — noting explicitly that no uniqueness is claimed, only that nothing downstream is tuned:

$$G_{ef} \;=\; \tfrac{1}{8}\,\mathrm{Tr}\big[\Omega^{-1}(\partial_e\Omega)\,\Omega^{-1}(\partial_f\Omega)\big]\,, \qquad \Omega = \sqrt{K}, \tag{4}$$

the fidelity susceptibility of the Gaussian ground state $\psi_0 \propto e^{-x^{\mathsf{T}}\Omega x/2}$ — positive by construction, nobody's choice. Equation (4) follows from the exact Gaussian overlap $|\langle\psi_1|\psi_2\rangle| = \det(\Omega_1)^{1/4}\det(\Omega_2)^{1/4}/\det(\frac{\Omega_1+\Omega}{2}$ we validated the implementation against central differences of this formula to 1.0×10^{-6}.

4 Attraction, exactly

With G from (4) as the cost, we measured the pair cost $V(d)$ of two opposite-parity consumers.

In $d = 1$ the measured $V(d)$ is the exact ring Green function to a fractional residual of 3×10^{-6} (Fig. 1), with V increasing with separation: attraction. In $d = 2, 3$, $E(c) = \frac{1}{2}c^{\mathsf{T}}W^{+}c$ makes the interaction exactly the Green function of W, whose symbol we computed in closed form (validated against the direct construction at 4.8×10^{-15}): W has an exact zero at the staggered point $q^* = (\pi, \ldots, \pi)$ — the emergent Gauss law — and quadratic dispersion around it (Fig. 2). The mechanism of the sign is transparent: the unsigned sum rule maps consumers to *staggered* charges $\sigma_i = (-1)^i c_i$; opposite-parity consumers are opposite Poisson charges of the positive-definite kernel W^+ and therefore attract, while same-parity consumers repel — a two-signed, electrostatics-like sign structure whose consequences we return to in the discussion. Nothing in the construction was tuned: the lattice, the sum rule, and (4) determine everything measured above.

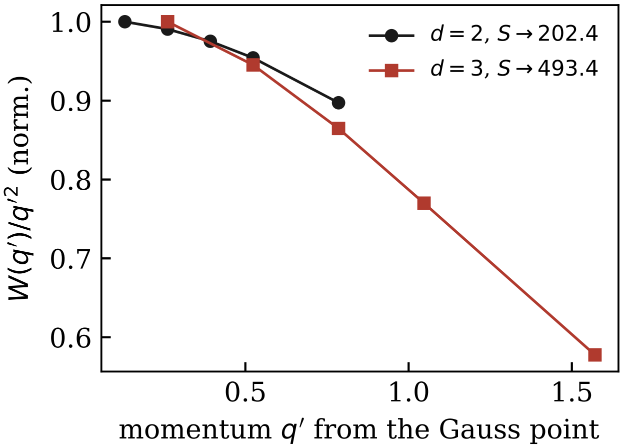

Figure 2: Two and three dimensions: the dispersion of the dressed operator W near its unique zero (the Gauss point), $W(q')/q'^2$, normalized to the smallest momentum. The finite limit establishes infrared Coulomb behavior; the limiting stiffness defines the dressed Newton constant $1/G_N$ (202.4 and 493.4 in lattice units for $d = 2, 3$; pre-registered gates passed at 0.24% and 0.36%).

5 The no-go theorem

Theorem 1 (drainage dichotomy)**.** *In translation-invariant quadratic drainage: (i) with the unsigned (bipartite) sum rule, the response kernel's only long-range pole lies at the staggered point q^*, and every local observable that is even under the sublattice swap carries a filter factor vanishing at q^*; consequently the long-range response of such observables is* quadratic *in the source — the linear response is confined to tidal (gradient-suppressed) and short-range terms, and in $d = 1$ vanishes identically. (ii) With the signed (divergence) sum rule, the like-pair interaction is $E_{\text{int}}(d) = f^2 K(d)$ with K the kernel of a positive operator: like sources repel for* every *admissible cost; and any local observable is a functional of nearby couplings — the flux — while the potential is a nonlocal integral of it.*

Proof sketch. (i) The first-order field response is $\widehat{\delta k}(q) = -\widehat{G}(q)^{-1}\widehat{A}(q)^\dagger \widehat{W}(q)^{-1}\widehat{c}(q)$; since $\widehat{G}$ is positive and bounded, poles occur only where $\widehat{A}(q) = 0$, i.e. $1 + e^{-iq_a} = 0\ \forall a$: the unique point q^*. A parity-neutral local observable acts at linear order through a filter whose symbol contains the factor $\prod_a (1 + e^{-iq_a})/2$, vanishing *at* q^*: the product of filter and pole is finite, and the surviving term carries one extra power of $(q - q^*)$ — one gradient of the envelope. At second order, squaring the staggered wave demodulates q^* to 0 and the envelope squared passes unfiltered: the leading long-range response is quadratic. In $d = 1$ the sum rule at empty sites forces exact staggering, and the linear response of cell-averaged observables vanishes identically (measured: 3×10^{-15} against a 3×10^{-4} quadratic part). (ii) $E(c) = \frac{1}{2} c^\top W^+ c$ with $W^+ \succeq 0$ gives $E_{\text{int}}(d) = f^2 K(d)$, K positive-definite; Bochner then bounds $K(0) \geq |K(d)|$ and the unique quadratic zero of $\widehat{W}$ at $q = 0$ makes K strictly decreasing at long range: energy released by separation. Locality of observables versus nonlocality of $\lambda = -W^+ c$ is definitional. Full details in the public record [10]. □

Corollary 1. *No theory in this class exhibits gravitational-redshift phenomenology (long-range*

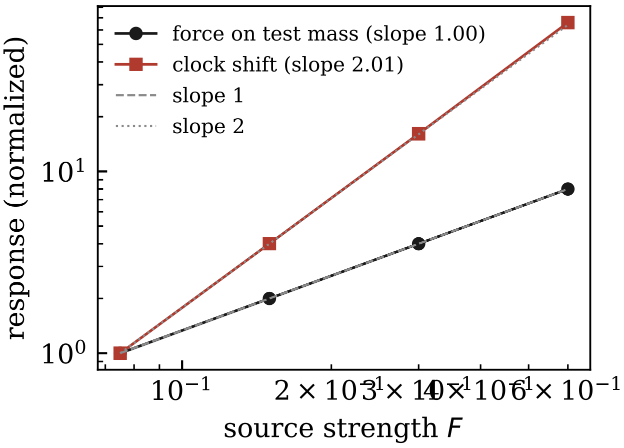

Figure 3: The dichotomy, measured: response of a test mass's *force* (slope 1.00) versus a cell-averaged *clock* observable (slope 2.01) to the same source, log-log in source strength. The linear channel exists for the force — matter is parity-charged and reads the pole — and is absent for parity-neutral clocks, exactly as Theorem 1(i) requires.

clock response, linear in the source, universal across clock constructions) together with attraction between like sources.

The theorem's mechanism, not merely its statement, is the point: the force survives because matter is parity-*charged* and couples to the pole directly; clocks fail because any parity-neutral functional filters the pole out. In general relativity both live in one object because the potential is itself a local field, g_{00}. The theorem therefore identifies the assumption that carries the metric in emergent-gravity derivations — Jacobson's local modular Hamiltonians [1] are precisely a local potential postulate — and shows it is not obtainable from conservation structure alone.

6 Inertia, the equivalence principle, and γ

The class supplies its own inertia. For a dressing whose drainage cloud is $\delta k^*(x)$, second-order adiabatic response gives, with no freedom,

$$M_{\rm ad} = 2\hbar^2 \sum_{n\neq 0} \frac{|\langle n|\partial_x\psi_0\rangle|^2}{E_n - E_0} = \frac{1}{4}\sum_{ij} \frac{\tilde{X}_{ij}^2}{\omega_i\omega_j(\omega_i+\omega_j)^3}, \tag{5}$$

the *gap-weighted* information metric ($\tilde{X}$: the cloud's translation generator in the mode basis; the second equality is the two-phonon evaluation, anchored on the exact single-mode result $M = \hbar(\partial_x\omega)^2/8\omega^3$ to 3×10^{-16}). Under $k\to\lambda k$: the cost curvature scales as λ^{-2} (the conformal weight of G), and $M_{\rm ad}\propto\lambda^{-5/2}$ (verified to 9×10^{-7}; independently re-derived in a hostile review). Charge, potential and inertia thus descend from one geometric object: the equivalence principle enters as an identity of scaling dimensions rather than a postulate.

If maintenance is priced per unit of the monitor's time, $V = \hbar\gamma_{\rm m}C$ with $\gamma_{\rm m}\propto k^a$, the operational clock $\omega^2 = V''/M_{\rm ad}$ carries $s = a/2+1/4$ per unit $\delta k/k$ against the substrate's

Claim	Gate	Result	Status
Energy cannot price (Lem. 1)	Hessian $\prec 0$; FD check	4×10^{-9}	pass
1D Coulomb (Fig. 1)	Green-fit residual	3×10^{-6}	pass
Gauss zero mode ($d = 2, 3$)	$\lvert W(q^*)\rvert / \max$	10^{-32}	pass
IR dispersion $d = 2$	ratio dev. $< 5\%$	0.24%	pass
IR dispersion $d = 3$ (replication)	ratio dev. $< 5\%$	0.36%	pass
Dichotomy, 1D linear response	$= 0$	3×10^{-15}	pass
Clock response quadratic	exponent	2.01	pass
Force response linear	exponent	1.00	pass
Inertia scaling $k^{-5/2}$	dev. $< 10^{-3}$	9×10^{-7}	pass
γ at a=0 / a=$\frac{1}{2}$	$\pm 1\%$	$+0.9999$ / -0.0000	pass

Table 1: Load-bearing pre-registered gates. Full registrations, scripts (each runs in minutes on a laptop), and the failures are public [10].

optical response of 1/2 (measured by wave propagation at the 10% level), giving, in the flat-space representation of the weak field [8],

$$\gamma \;=\; \frac{1-2a}{1+2a}. \tag{6}$$

Measured: $a = 0$ yields $\gamma = +0.9999$; $a = \frac{1}{2}$ yields $\gamma = -0.0000$. The result holds in a strong-monitoring regime ($\gamma_{\mathrm{m}} F \gtrsim 7$, a measured scaling law) and under assumptions inventoried in the record (multiplicative pricing; local-density bridge; the identification of light with substrate waves; the overdamping caveat) — we state them because the formula is only as strong as they are. On the minimal reading of the monitoring axiom — the audited violations are dimensionless bookkeeping residing at exactly zero frequency — $a = 0$ is forced, and Cassini's $|\gamma - 1| < 2.3 \times 10^{-5}$ [9] reads as $|a| < 5.8 \times 10^{-6}$ — with the caveat that the optical ingredient of the chain (the substrate response 1/2) is verified here only at the 10% level by direct wave propagation; the reading's precision is Cassini's, its derivation's is not yet.

7 Numerical protocol and gates

Every claim above was pre-registered: gates and kill criteria committed to a public repository before the corresponding code existed. Table 1 collects the load-bearing gates. Twenty-seven predecessor claims — including three of our own clock constructions and an earlier hand-declared cost whose "measured attraction" failed reproduction — were retired by these methods and are published at equal prominence [10].

8 Discussion

What is claimed. A constructive result — the sign of an interaction derived from a fidelity susceptibility under a conservation constraint — and a no-go theorem locating the metric's true origin outside conservation structure. The two halves reinforce: the construction shows how much gravity-like structure conservation *can* produce (Gauss law, $1/r^{d-1}$, a dressed coupling, an equivalence-principle identity), and the theorem shows exactly where it must stop (the clock sector), and why (no local potential field).

What is not claimed. No physical units; no dynamical (radiative, gravitomagnetic, or nonlinear) sector — the solar system's precision tests of those sectors are, on our own analysis, fatal to any *static* member of this class taken as a complete theory of gravity. The two-signed charge structure of the conserved sector suggests an electromagnetic rather than gravitational reading of the constructive half — indeed the no-go theorem is then simply this sector announcing its gauge invariance, which resolves the apparent tension between the present paper's framing (a gravity no-go) and that reading (the sector was never gravity): both are the same statement seen from two sides. We develop the reading, its checks, and a falsifiable decoherence signature elsewhere [10], and keep the present paper to the two results that are theorems.

Relation to prior work. Attraction from a local conservation constraint appears, with a different engine, in Pretko's fracton gravity [4]; the information metric appears as a holographic *dual* in [5]; the quantum-metric effective mass is established condensed-matter physics [6, 7]. To our knowledge, the derivation of an interaction's *sign* from the fidelity susceptibility, the dichotomy theorem, and the $k^{-5/2}$ inertia identity are new; our public prior-art sweeps (with search protocols) accompany the record [10].

9 Methods and provenance

Human–AI collaboration: derivations, implementations and drafting by the AI under the direction of a human collaborator who prefers to remain unnamed. Adversarial review was performed by the AI operating persona-referees (a constructive and a hostile pass); this is a declared methodological device, *not* independent peer review — one of its findings (a discarded term 685× the kept one) materially amended §6 and is recorded. All code, registrations, failures and reviews: [10].

References

[1] T. Jacobson, *Entanglement equilibrium and the Einstein equation*, PRL 116, 201101 (2016).

[2] T. Faulkner et al., JHEP 03 (2014) 051.

[3] E. Verlinde, SciPost Phys. 2, 016 (2017).

[4] M. Pretko, *Emergent gravity of fractons*, PRD 96, 024051 (2017).

[5] M. Miyaji et al., PRL 115, 261602 (2015).

[6] J. P. Provost, G. Vallée, Commun. Math. Phys. 76, 289 (1980).

[7] P. Törmä, S. Peotta, B. Bernevig, Nat. Rev. Phys. 4, 528 (2022).

[8] R. H. Dicke, Rev. Mod. Phys. 29, 363 (1957).

[9] B. Bertotti, L. Iess, P. Tortora, Nature 425, 374 (2003).

[10] The full record: `https://emergent-gravity.com` (registrations, scripts, twenty-seven retired claims, review reports).